

THE THEORY AND PRACTICE OF GENERAL ANALYSIS LIFT CALCULATIONS

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ABSTRACT

The formulae presented in this paper allow analysis of any peak passenger traffic flow for any practical configuration of lifts. This means that we can consider not only a typical office morning up peak, but also shopping centres, office blocks with underground car parks and roof top restaurants, etc - scenarios not dealt with by traditional lift traffic analysis methods. Derivation of the formulae and their implementation on a personal computer are discussed. Practical applications are demonstrated using the Oasys LIFT program which utilises the General Analysis formulae. Examples are taken from projects on which the author has advised.

1 INTRODUCTION

Many engineers designing with the help of computers have a healthy scepticism of complex computer models which sometimes give unrealistically precise predictions of our imprecise world.

But it is possible and worthwhile to learn about the effects on lift service of any passenger lift traffic flows where they can be predicted for a new building or measured in an existing building.

Analytical lift calculation procedures generally address specific passenger traffic flows. An up peak analysis describes the morning incoming peak in an office building, a down peak the reverse. A two way peak describes a combination of up and down peak traffic (eg the traffic flow in a multistorey car park). The interfloor analysis allows passengers to arrive at any level, but assumes their destinations are always in proportion to the individual floor populations. Each of these traffic flows are solely special cases for the General Analysis procedure described in this paper which, when implemented on a computer allows analysis of any peak passenger flow for any practical configuration of lifts.

Before considering the General Analysis formulae we must first define our measure of passenger traffic. The term commonly applied in an up peak analysis, the Five Minute Ratio, cannot describe fully the traffic we wish to define.

2 DEFINING LIFT PASSENGER TRAFFIC

The author's introduction to lift traffic analysis was through writing a lift simulation program, as noted in the biography of this paper. Any analytical analysis for the general case must define lift passenger traffic in a similar way to simulation models, identifying the rate of passenger arrivals at each lift landing station and the passengers' probable destinations. To define the passenger flow completely we need two terms:

u_i the passenger arrival rate at floor i (defined for each floor at which passenger may arrive)

d_{ij} the probability of destination floor of passenger from floor i being the j th floor (defined for all possible i and j)

It is easier to show these terms diagrammatically. For example the simple case of an up peak traffic flow in an office block could be as shown in figure 1.

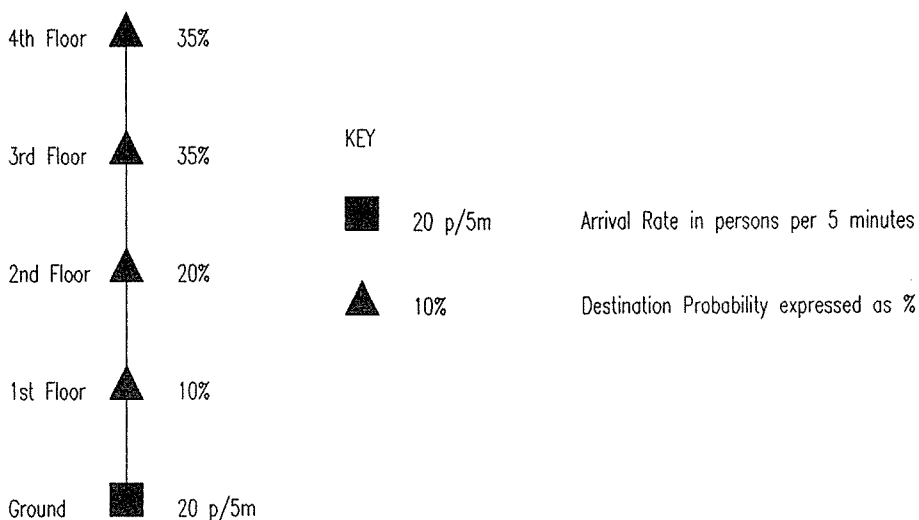


Figure 1 Example passenger traffic flow for a typical office up peak

A more complex traffic flow such as may be found in a shopping centre with car park above could be as shown in figure 2.

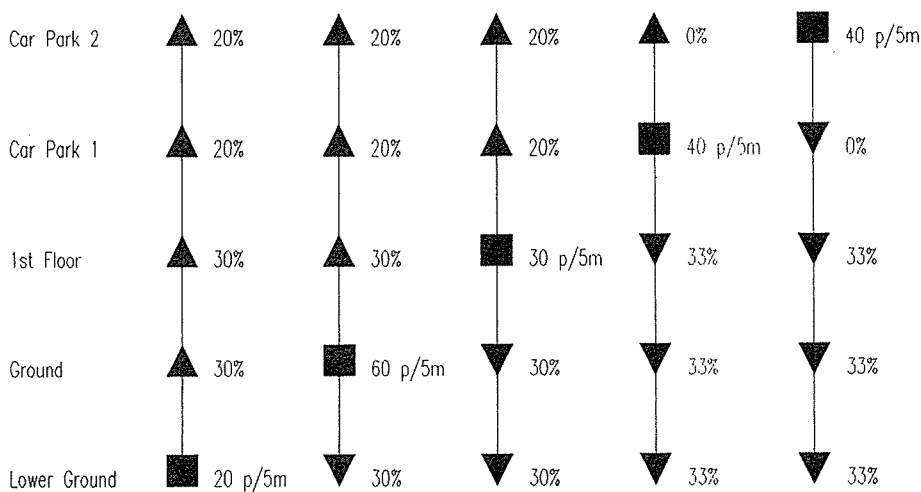


Figure 2 Example Passenger Traffic Flow for Shopping Centre with Car Park above

3 PRINCIPLES OF DERIVATION

Having defined passenger traffic we can apply our mathematical model. It is not the intension of this paper to give a line by line derivation of the formulae, but to establish the basic principles and assumptions. The complete set of formulae and definitions are given in the appendix to this paper. The formulae were fist published by the CIBSE Building Services Engineering Research and Technology Journal in 1990¹.

Before we write down the formulae we must define one more term:

$p_{ij}(n)$ the probability of n passenger travelling from the ith to the jth floor in the time interval T

It is generally accepted that the arrival of passengers at a lift landing station is reasonably approximated by a Poisson process². This gives us the result:

$$p_{ij}(n) = \frac{(\mu_i T d_{ij})^n}{n!} \cdot e^{-\mu_i T d_{ij}}$$

When dealing with probabilities it is generally easier to calculated the probability of an event not happening, and subtract that result from 1 to find the probability of the event. So, if we let

$$p_{ij} = p_{ij}(0)$$

we get:

$$p_{ij} = e^{-\mu_i T d_{ij}}$$

The probability of the lift stopping at the jth floor on its journey up is:

- 1- (probability of no calls from lower floors to the jth floor) * (probability of no calls from the jth floor to higher floors)

Mathematically, we can express this:

$$pUS_j = \left\{ 1 - \prod_{a=1}^{j-1} p_{aj} \prod_{j+1}^N p_{jb} \right\} \quad \text{for } \{2 \leq j \leq N-1\}$$

Applying this and similar principles we can derive

- S the probable number of stops in a round trip
- L the probable lowest reversal floor
- H the probable highest reversal floor

For more complex problems where lifts do not serve all the same floors, or are of different speeds, capacities, etc we must define "overlapping zones" where passengers are likely to take the first lift that arrives and serves their destination floor. The formulae allow us to calculate the split of traffic between overlapping zones for each possible journey from the ith to the jth floor.

4 COMPUTER IMPLEMENTATION

A standard IBM PC or compatible has sufficient computing power to calculate a solution for a typical lift configuration within seconds.

Implementation of the method is relatively straightforward for someone with experience of lift traffic analysis and computer programming. As is the case for many analytical programs, writing the user input/output routines can be the most difficult and time consuming task.

The calculation is iterative as the equations for probable number of stops and reversal floors are dependant on the Waiting Interval and vice-versa. The procedure here is to "guess" the Waiting Interval, revising the guess for each iteration until the guess equals the result.

The flow chart in figure 3 defines the main program structure and subroutines.

A further iteration is required if overlapping zones are defined as the calculation for splitting data between zones is dependant on the Waiting Interval of each zone and vice-versa.

5 THE OASYS LIFT PROGRAM

The Oasys (Ove Arup Computing Systems) LIFT program has been used on a wide range of projects throughout the international Ove Arup Partnership since its first issue in 1989.

The program is written in Fortran 77 with the help of Oasys standard input/output routines and run on PC's.

In addition to basic implementation of the General Analysis method, the program offers the user additional features:

- i) the computer automatically calculates destination probabilities for specific passenger traffic flows such as up peak, down peak, two way and interfloor when selected by the user
- ii) for the defined passenger traffic, the program will allow analysis over a range of possible solutions in a single analysis run (eg 3 to 4 lifts, 1.6 to 2.5m/s, etc)
- iii) a traditional up peak analysis calculation procedure is included and allows users to compare results and confirm consistency before considering more complex traffic flows with the General Analysis method

From the users viewpoint there is one main difference between using the traditional (OLD) and General (NEW) calculation procedure for an up peak analysis: in a traditional analysis you would normally assume that the lift was 80% full and calculate the resultant Five Minute Ratio. Using the General Analysis method the Five Minute Ratio is defined and the program determines how full the lift is as a result of this passenger traffic.

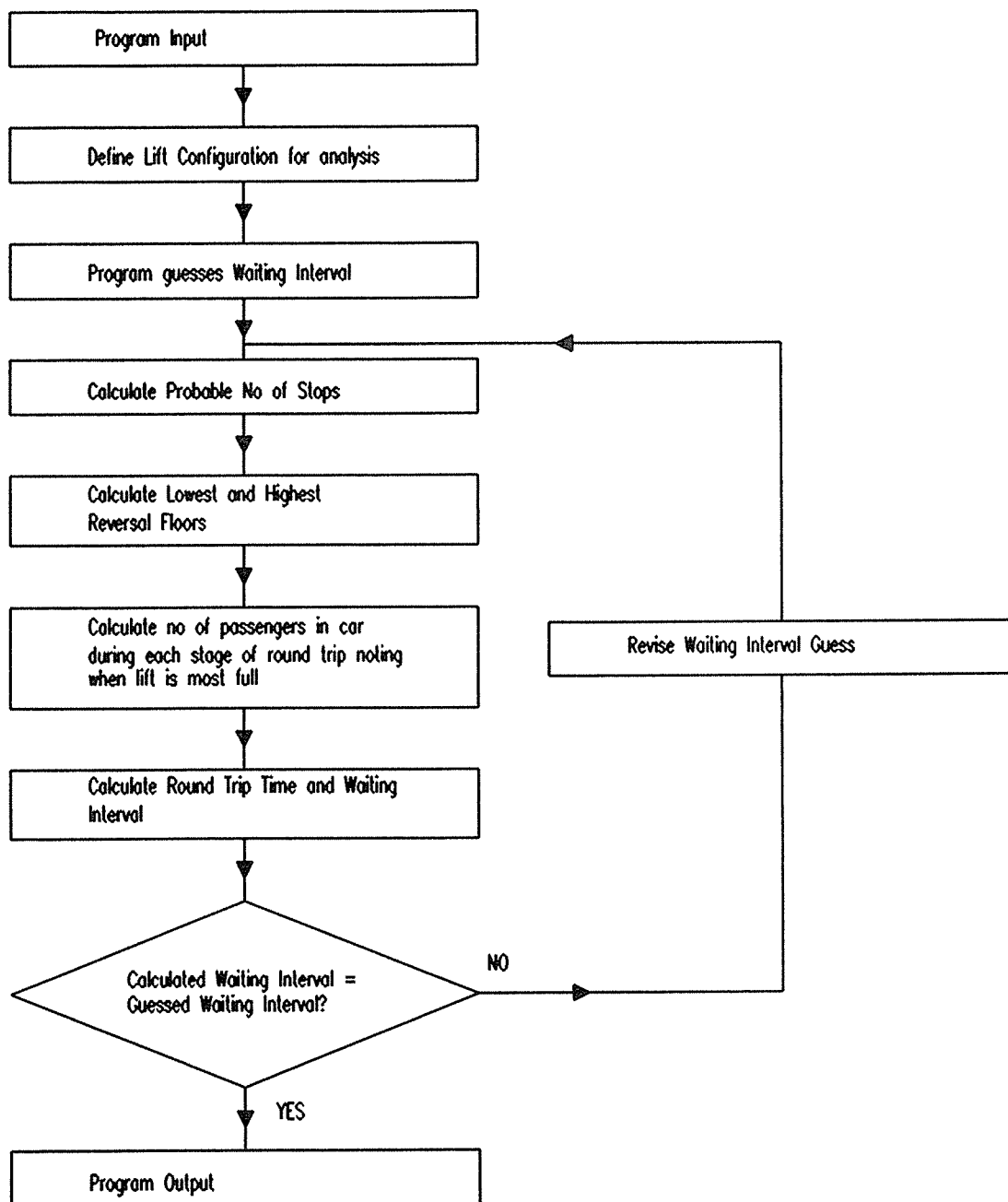


Figure 3 Program Flow Chart

This second approach is more helpful as specifiers often choose larger lifts than required for reasons of prestige, etc which if assumed 80% full give unrealistically high Five Minute Ratios and Waiting Intervals. The General Analysis just notes how full the lift is for the given Five Minute Ratio.

One occasional user problem is that zero Waiting Interval and Capacity Factor results are given when the program is asked to analyze an over-lifted solution. If there is insufficient traffic to keep the lifts busy, in effect there will normally be a lift left waiting at the main terminal floor unused - the iterative value of Waiting Interval tends to zero, as would be the case in real life.

6 EXAMPLES

6.1 A typical office up peak analysis

A prestigious office block in London with ground floor and eight upper stories. Floor populations range between 50 and 100 persons per floor. A stair factor of 50% (50% walk one floor, 25% two, 13% three, etc) has been assumed.

The morning up peak is likely to be the busiest time for the lifts. A Five Minute Ratio of 17% is assumed based on the possibility of an unified tenancy with the majority of employees starting work in the morning at approximately the same time.

The user enters the data, selecting the up peak analysis (so not needing to define destination probabilities which are in proportion to the population of each office floor). The lift analysis selection criteria is set over a range of numbers of lifts, capacities, speeds, etc with limits defined for the maximum Waiting Interval and Capacity Factor acceptable.

Each configuration is analyzed, rejected or displayed dependant on whether or not the selection criteria are met. The user selects his preferred solution from the output, in this case:

No of lifts	3
Contract Speed	2.0m/s
Lift Capacity	1000kg
Probable No of Stops	6.4
Probable Highest Reversal floor	7
Waiting Interval	30s
Capacity Factor	70%

For this "simple" case the user could compare results with the alternative, traditional up peak analysis. Here the default Capacity Factor is 80% and the Five Minute Ratio is calculated as opposed to input. The results are as follows:

No of lifts	3
Contract Speed	2.0m/s
Lift Capacity	1000kg
Waiting Interval	33s
Five Minute Ratio	18%

The results are very similar for both analysis methods. The second set of results have a marginally higher Waiting Interval and Five Minute Ratio, but this corresponds with the lift being 80% as opposed to 70% full for the round trip.

Having demonstrated consistency for an up peak analysis between the General Analysis and a traditional calculation method, we will now consider a more complex traffic flow.

6.2 Lunchtime peak at office block with roof top restaurant

Again a prestigious office block (with lower ground, ground and 6 upper floors), but this time the architect has suggested locating the office restaurant/cafeteria on the top floor. The lifts are satisfactory to cope with the morning up peak, but will they cope with the a lunchtime peak?

In this case the lunchtime peak is likely to be made up of a combination of:

- i) passengers travelling from their offices to the roof top restaurant for lunch
- ii) passengers travelling back to their offices after lunch at the restaurant
- iii) passengers travelling to the ground floor to leave the building to buy sandwiches or eat out
- iv) passengers returning from buying/eating lunch out

The estimate of the combined traffic flows early during the lunch hour was as shown in figure 4 (less people are expected to travel within a five minute period than during the morning up peak).

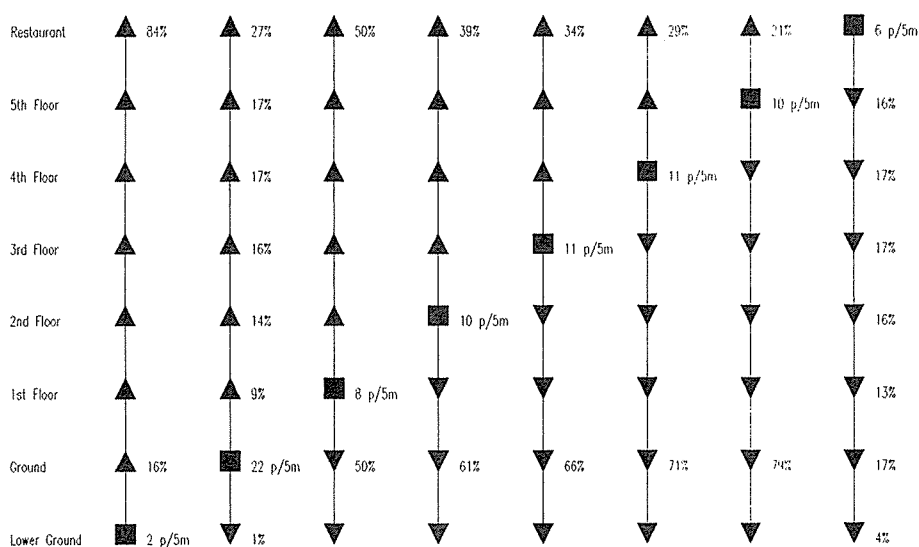


Figure 4 Summation of lunchtime peak passenger flows for office block with restaurant on top floor

The analysis gives the following results:

No of lifts	3
Contract Speed	2m/s
Waiting Interval	34s
Capacity Factor (1250kg lift)	23%
Probable No of stops	8.6
Probable Lowest Reversal Floor	G
Probable Highest Reversal Floor	Restaurant

The lifts always reach the top floor during their round trip, unlike during the up peak. Lifts are also stopping regularly during their journey down rather than expressing to the ground floor, another delay.

In this case the lifts can cope with the specified traffic flow, but the service is poorer than during the morning up peak.

7 CURRENT DEVELOPMENTS

The theory has been extended to incorporate double decker lifts and is currently being implemented within the Oasys LIFT program. The formulae will be published in due course.

8 CONCLUSIONS

The General Analysis gives us a tool to analyze any peak passenger traffic flow quickly and effectively. Analytical as opposed to the more complex and computer power-hungry simulation models are likely to be our basic lift traffic analysis tools for the foreseeable future. The writer proposes this General Analysis method for implementation as a total solution for analysis of all peak passenger traffic flows.

We need to define clearly expected passenger traffic before embarking on a detailed analysis, and interpret results remembering the limited accuracy of our input data. There appears to be little published data detailing surveys of lift passenger traffic flows - this needs to be addressed if we are to make full use of our new passenger traffic analysis tools.

REFERENCES

1. Peters R D CIBSE Building Services Engineering Research and Technology Volume 11 Number 2 (1990)
2. Barney G C and dos Santos S M Elevator Traffic Analysis Design and Control 2nd edn (London: Peter Peregrinus)(1985)

APPENDIX - General Analysis Formulae

A List of symbols

μ_i	Passenger arrival rate, floor i (persons s ⁻¹)
d_{ij}	Probability of the destination floor of a call from i being the jth floor
T	Waiting Interval (s)
N	Number of floors
$P_{ij}(n)$	Probability of n passengers travelling from the ith to the jth floor in the time interval, T
P_{ij}	Probability of no calls from the ith to the jth floor in the time interval, T
pS_1	Probability that a lift will stop at the lowest floor
$pUS_2, pUS_3, \dots, pUS_{N-1}$	Probability that the lift will stop each of the intermediate floors on its journey up
pS_N	Probability that the lift will stop at the highest floor, N
pDS_{N-1}, \dots, pDS_2	Probability that the lift will stop at each of the intermediate floors on its journey down
S	Probable number of stops
pH_n	Probability of nth floor being the Highest Reversal Floor
pL_n	Probability of nth floor being the Lowest Reversal Floor
H	Highest Reversal Floor
L	Lowest Reversal Floor
$JWI(i, j)$	Journey waiting interval for passengers travelling from the ith to the jth floor (s)
$T(\text{Zone } n)$	Waiting interval, zone n (s)

Split(Q, i, j) Proportion of passengers travelling from the ith to the jth floor who are using lifts in zone Q

B Call Probabilities

$$p_{ij}(n) = \frac{(\mu_i T d_{ij})^n}{n!} \cdot e^{-\mu_i T d_{ij}} \tag{1}$$

$$p_{ij} = e^{-\mu_i T d_{ij}} \tag{2}$$

C Probable number of stops

$$pS_1 = \left\{ 1 - \prod_{a=2}^N p_{a1} \prod_{b=2}^N p_{1b} \right\} \tag{3}$$

$$pUS_j = \left\{ 1 - \prod_{a=1}^{j-1} p_{aj} \prod_{j+1}^N p_{jb} \right\} \quad \text{for } \{2 \leq j \leq N-1\} \tag{4}$$

$$pS_N = \left\{ 1 - \prod_{a=1}^{N-1} p_{aN} \prod_{b=1}^{N-1} p_{Nb} \right\} \tag{5}$$

$$pDS_j = \left\{ 1 - \prod_{a=j+1}^N p_{aj} \prod_{b=1}^{j-1} p_{jb} \right\} \quad \text{for } \{2 \leq j \leq N-1\} \tag{6}$$

$$S = pS_1 + \sum_{j=2}^{N-1} \{ pUS_j + pDS_j \} + pS_N \tag{7}$$

D Highest Reversal Floor

$$pH_1 = \prod_{a=1}^N \prod_{b=1}^N p_{ab} \tag{8}$$

$$pH_j = \left\{ 1 - \prod_{a=1}^{j-1} p_{aj} \prod_{b=1}^{j-1} p_{jb} \right\} \times \left\{ \prod_{a=1}^N \prod_{b=j+1}^N p_{ab} \prod_{a=j+1}^N \prod_{b=1}^j p_{ab} \right\} \tag{9}$$

for $\{2 \leq j \leq N-1\}$

$$pH_N = \left\{ 1 - \prod_{a=1}^{N-1} p_{aN} \prod_{b=1}^{N-1} p_{Nb} \right\} \tag{10}$$

(a good check for this is $\sum_{j=1}^N pH_j = 1$)

$$H = \sum_{j=1}^N j p_{H_j} \quad (11)$$

E Lowest Reversal Floor

$$p_{L_1} = \left\{ 1 - \prod_{a=2}^N p_{a1} \prod_{b=2}^N p_{1b} \right\} \quad (12)$$

$$p_{L_j} = \left\{ 1 - \prod_{a=j+1}^N p_{aj} \prod_{b=j+1}^N p_{jb} \right\} \times \left\{ \prod_{a=1}^{j-1} \prod_{b=1}^{j-1} p_{ab} \prod_{a=1}^{j-1} \prod_{b=j}^N p_{ab} \right\} \quad (2 \leq j \leq N-1) \quad (13)$$

$$p_{L_N} = \prod_{a=1}^N \prod_{b=1}^N p_{ab} \quad (14)$$

$$L = (N+1) - \sum_{j=1}^N p_{L_j} ((N+1) - j) \quad (15)$$

F Passengers in car calculation - eg ith floor journey up

$$T \mu_i \sum_{j=i+1}^N d_{ij} \text{ passengers enter the car}$$

$$T \sum_{j=1}^{i-1} \mu_j d_{ji} \text{ leave the car}$$

G Overlapping Zones

$$JWI(i, j) = \frac{1}{\sum_{N=\{Z\}} \frac{1}{T(N)}} \quad (16)$$

{Z} = {all zones serving both the ith and the jth floor}

$$\text{split } (Q) = \frac{JWI(i, j)}{T(Q)} \quad (17)$$

BIOGRAPHY

Richard Peters graduated in 1987 from the University of Southampton, England with a BSc Hons in Electrical Engineering. He subsequently joined Ove Arup & Partners (Consulting Engineers) in London who had sponsored him during his time at University. His interest in lift traffic analysis began at Southampton University where he wrote a lift simulation program as part of his final year project. Since joining Arup, he has concentrated on developing analytical lift traffic analysis models. He is the principle author of the Oasys (Ove Arup Computing Systems) LIFT program which is used regularly on a wide range of projects to select suitable lift configurations.